Journal of Namibian Studies, 35 S1 (2023): 1239-1246 ISSN: 2197-5523 (online)

A New Notion Of Mappings In Nano Ideal Topological Spaces

Dr. J. Arul Jesti[1] and J. Joycy Renuka[2]

[1]Assistant Professor
[2]Research Scholar
St. Mary's College (Autonomous)
Thoothukudi-628001, India

ABSTRACT

By M. Parimala [6] et al., the concept of nano ideal topological space was first presented. They researched its characterizations and characteristics. Additionally, they explored some of the fundamental features of the notion of nano ideal generalised closed sets in nano ideal topological spaces. Following that, several mathematicians created different types of closed sets, closed maps, and homeomorphism, including nIg-closed maps [4], nIg -closed mappings [4], and nIg -homeomorphism [4] in nano ideal topological spaces. In the current study, we developed a unique concept to depict two types of mappings: $\mathrm{nIgsemi}^*$-closed maps and $\mathrm{nIgsemi}^*$-open maps. Additionally, we looked at how these mappings relate to the closed and open maps that currently exist in nano ideal topological spaces.

Key words: $\mathrm{nIgsemi}^*$-closed map, $\mathrm{nIgsemi}^*$-open map.

1.Preliminary

Definition 1.1 [5] A function $f\colon (U, \tau_R(X)) \to (V, \tau_R(Y))$ is defined to be **nano open (closed) function** if every nano open (closed) set A of U is mapped to a nano open (closed) set in V.(Lellis Thivagar, M & Carmel Richard 2013)

Definition 1.2 [4] A map $f\colon (O, N, I) \to (P, N', J)$ is called

(1) $\mathbf{n}^*$**-closed** if for every n-closed subset G of (O, N, I), $f(G)$ is n^*-closed in (P, N', J).

(2) $\mathbf{nIg}$**-closed** (resp. nIg-open) map if for every n-closed (resp. n-open) subset G of (O, N, I). $f(G)$ is nIg-closed (resp. nIg-open) in (P, N', J) (Ganesan, S, 2020).

Definition 1.3 [3] A mapping $f\colon (U, \tau_R(X)) \to (V, \tau_{R^*}(Y))$ is called **nano g–closed**

if the image of every nano g–closed set in $(U,\tau_R(X))$ is nano g–closed in $(V,\tau_{R^*}(Y))$ (Bhuvaneswari, K & Mythili Gnanapriya, K 2014).

Definition 1.4 [5] Let $(U\,\tau_R(X))$ and $(V,\tau_{R_0}(X))$ be nano topological spaces. A mapping $f:(U,\tau_R(X)) \to (V,\tau_{R_0}(X))$ is said to be **nano continuous** if the inverse image of every nano open set in $(V,\tau_{R_0}(X))$ is nano open set in $(U,\tau_R(X))$ (Lellis Thivagar, M & Carmel Richard 2013).

Definition 1.5 [1] A subset A of a nano ideal topological space (U,N,I) is said to be **nano ideal generalized semi^*-closed** (briefly, $\mathrm{nIgsemi}^*$-closed) if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is nano semi^*-open. The family of all $\mathrm{nIgsemi}^*$-closed sets of U is denoted by $\mathrm{nIgsemi}^*C(U,N,I)$(or simply $\mathrm{nIgsemi}^*C(U)$ **(**Arul Jesti, J & Joycy Renuka, J, 2020).

Theorem 1.6 [1] Every nano closed set is $\mathrm{nIgsemi}^*$-closed but not conversely.

Theorem 1.7 [1] If (U,N,I) is any nano ideal topological space, then every ng-closed set is $\mathrm{nIgsemi}^*$-closed but not conversely.

Theorem 1.8 Let (U,N,I) be a nano ideal topological space. Every n^*-closed set is $\mathrm{nIgsemi}^*$-closed [1].

Definition 1.9 A subset A of a nano ideal topological space (U,N,I) is said to be **nano ideal generalized semi*-open** (briefly, $\mathrm{nIgsemi}^*$-open) if $U - A$ is $\mathrm{nIgsemi}^*$-closed. The family of all $\mathrm{nIgsemi}^*$-open sets of U is denoted by $\mathrm{nIgsemi}^*O(U,N,I)$(or simply $\mathrm{nIgsemi}^*O(U)$) [1]

Definition 1.10 [2] A map $f:(U,N,I) \to (V,M,J)$ is called **$\mathbf{nIgsemi}^*$-continuous function**, if $f^{-1}(A)$ of each n-open set $A \subseteq (V,M,J)$ is a $\mathrm{nIgsemi}^*$-open set in (U,N,I). (Arul Jesti, J & Joycy Renuka, J, 2022).

2. $\mathbf{nIgsemi}^*$-Closed Maps

Definition 2.1 A map $f:(U,N,I) \to (V,M,J)$ is said to be **$\mathbf{nIgsemi}^*$-closed map** if for every nano closed (n-closed) subset G of (U,N,I), $f(G)$ is $\mathrm{nIgsemi}^*$-closed in (V,M,J).

Example 2.2 Consider the universal set $U = \{1,2,3,4\}$, the approximation space $U/\mathcal{R} = \{\{1\},\{2\},\{3,4\}\}$, $X = \{\{2\},\{4\}\} \subseteq U$ with the ideal $I = \{\varphi,\{1\}\}$. The nano topology defined by U is $\tau_R(X) = \{U,\varphi,\{2\},\{3,4\},\{2,3,4\}\}$ and $\mathrm{nIgsemi}^*$-closed sets are $\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,3,4\},\{1,2,4\},U,\varphi\}$. Assume $V = \{1,2,3,4\}$, the approximation space $V/\mathcal{R} = \{\{1\},\{3\},\{2,4\}\}$, $Y = \{1,2\} \subseteq V$ with the ideal $J = \{\varphi,\{1\}\}$. The nano topology defined by V is $\Omega_R(Y) = \{V,\varphi,\{1\},\{2,4\},\{1,2,4\}\}$ and $\mathrm{nIgsemi}^*$-closed sets are $\{\{1\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},V,\varphi\}$.

Then $f:(U,N,I)\to(V,M,J)$ is defined as $,f(4)=4, f(1)=2$, $f(3)=3, f(2)=1$ is nIgsemi*-closed map.

Theorem 2.3 A map $f:(U,N,I)\to(V,M,J)$ is nIgsemi*-closed iff for any subset S of (V,M,J) and for each nano open set P of (U,N,I) contains $f^{-1}(S)$, there exist an nIgsemi*-open set O of (V,M,J) such that $S\subseteq O$ and $f^{-1}(O)\subseteq P$.

Proof. Necessity: Let P be n-open in (U,N,I) and S be any subset of (V,M,J) . Then P^c is n-closed in (U,N,I). Since f is an nIgsemi*-closed map, $f(P^c)$ is nIgsemi*-closed in (V,M,J). Thus $V-f(P^c)$ is nIgsemi*-open, say O containing S such that $f^{-1}(O)=f^{-1}\big(V-f(P^c)\big)=U-P^c=P$. Thus $f^{-1}(O)\subseteq P$.

Sufficiency: Let's assume F be a nano closed set in (U,N,I) and S be any subset of (V,M,J). Put $S=(f(F))^c$. Then we have $f^{-1}(S)\subseteq F^c$ and F^c is n-open in (U,N,I). By hypothesis, there exits an nIgsemi*-open set O of (V,M,J) such that $S\subseteq O$, and $f^{-1}(O)\subseteq F^c$. Now $f^{-1}(O)\subseteq F^c$ implies $F\subseteq (f^{-1}(O))^c=f^{-1}(O^c)$. That is $f(F)\subseteq O^c$---------(1). Now $S\subseteq O$ implies $(f(F))^c\subseteq O$. Then $O^c\subseteq f(F)$----------(2). From (1) and (2), $f(F)=O^c$. Since O^c is nIgsemi*-closed, f(F) is nIgsemi*-closed in (V,M,J). Hence f is nIgsemi*-closed.

Theorem 2.4 If $f:(U,N,I)\to(V,M,J)$ is a nano closed map, then it is nIgsemi*-closed map.

Proof. Let G be a n-closed subset of (U,N,I). Then f(G) is n-closed in (V,M,J) because f is a n-closed map. Now f(G) is nIgsemi*-closed in (V,M,J) because every n-closed is nIgsemi*-closed. Hence f is nIgsemi*-closed map.

Theorem 2.5 If $f:(U,N,I)\to(V,M,J)$ is a n^*-closed map, then the function f is nIgsemi*-closed map.

Proof. Let G be a n-closed subset of (U,N,I). Then f(G) is n^*-closed in (V,M,J) because f is a n^*-closed map Now f(G) is nIgsemi*-closed in (V,M,J) because every n^*-closed is nIgsemi*-closed. Hence f is nIgsemi*-closed map.

Remark 2.6 A nIgsemi*- closed map may not be a n^*-closed map. The following example explains this remark.

Example 2.7 Let U, N, I, V, M, J and f be defined as an example 2.2. Then n^*-closed sets are $\{\{1\},\{3\},\{1,3\},\{2,3,4\},V,\varphi\}$. Here f is nIgsemi*-closed map but not n^*-closed map because $f(\{1,2\})=\{1,2\}$ is not n^*-closed in (V,M,J).

Theorem 2.8 If $f:(U,N,I)\to(V,M,J)$ is a nIgsemi*-closed map, then it is nIg-closed map.

Proof. Let G be a n-closed subset of (U,N,I). Then f(G) is nIgsemi*- closed in (V,M,J) because f is a nIgsemi*-closed map. This implies f(G) is nIg-closed in (V,M,J) because every nIgsemi*-closed is nIg-closed. As a result, f is nIg-closed map.

Remark 2.9 A nIg-closed map may not be a $\mathrm{nIgsemi}^*$-closed map.
The following example explains this remark.

Example 2.10 Assume $\mathrm{U} = \{1,2,3,4\}$, the approximation space $\mathrm{U}/\mathcal{R} = \{\{1\},\{3\},\{2,4\}\}$, $\mathrm{X} = \{1,2\} \subseteq \mathrm{U}$ with the ideal $\mathrm{I} = \{\varphi,\{1\}\}$. The nano topology defined by U is $\tau_{\mathrm{R}}(\mathrm{X}) = \{\mathrm{U},\varphi,\{1\},\{2,4\},\{1,2,4\}\}$ and $\mathrm{nIgsemi}^*$- open sets are $\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,4\},\{2,3,4\},\mathrm{U},\varphi\}$. Let $\mathrm{V} = \{1,2,3,4\}$, the approximation space $\mathrm{V}/\mathcal{R} = \{\{3\},\{4\},\{1,2\}\}$, $\mathrm{Y} = \{\{2\},\{4\}\} \subseteq \mathrm{V}$ with the ideal $\mathrm{J} = \{\varphi,\{1\}\}$. The nano topology defined by V is $\Omega_{\mathrm{R}}(\mathrm{Y}) = \{\mathrm{V},\varphi,\{3\},\{4\},\{1,2,4\}\}$ and $\mathrm{nIgsemi}^*$-closed sets are $\{\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\},\{1,4\}\{2,4\},\{1,2,3\},\{2,3,4\},\{1,2,4\},\{1,3,4\},\mathrm{V},\varphi\}$.
nIg-closed sets are $\{\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{3,4\},\{1,3\},\{1,4\}\{2,4\},\{1,2,3\},\{2,3,4\},\{1,2,4\},\{1,3,4\},\mathrm{V},\varphi\}$.
Then $\mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \to (\mathrm{V},\mathrm{M},\mathrm{J})$ is defined as $,\mathrm{f}(4) = 1$, $\mathrm{f}(3) = 3, \mathrm{f}(2) = 2, \mathrm{f}(1) = 4$ is nIg- closed map. But the function f is not $\mathrm{nIgsemi}^*$-closed map since $\mathrm{f}(\{1,3\}) = \{3,4\} \notin \mathrm{nIgsemi}^*\mathrm{C}(\mathrm{U},\mathrm{N},\mathrm{I})$.

Theorem 2.11 If $\mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \to (\mathrm{V},\mathrm{M},\mathrm{J})$ is nano closed and $\mathrm{g}\colon(\mathrm{V},\mathrm{M},\mathrm{J}) \longrightarrow (\mathrm{W},\mathrm{O},\mathrm{K})$ is $\mathrm{nIgsemi}^*$-closed, then $\mathrm{g} \circ \mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \longrightarrow (\mathrm{W},\mathrm{O},\mathrm{K})$ is $\mathrm{nIgsemi}^*$-closed.

Proof: Let's assume G is any nano closed set in U. Then obviously $\mathrm{f}(\mathrm{G})$ is nano closed in V because $\mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \to (\mathrm{V},\mathrm{M},\mathrm{J})$ is nano closed map. Also $\mathrm{g}(\mathrm{f}(\mathrm{G}))$ is $\mathrm{nIgsemi}^*$-closed in W because $\mathrm{g}\colon(\mathrm{V},\mathrm{M},\mathrm{J}) \longrightarrow (\mathrm{W},\mathrm{O},\mathrm{K})$ is $\mathrm{nIgsemi}^*$-closed. Since $(\mathrm{g} \circ \mathrm{f})(\mathrm{G}) = \mathrm{g}\big(\mathrm{f}(\mathrm{G})\big)$, $\mathrm{g} \circ \mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \longrightarrow (\mathrm{W},\mathrm{O},\mathrm{K})$ is $\mathrm{nIgsemi}^*$-closed.

3. $\mathbf{nIgsemi}^*$-Open Maps

Definition 3.1 A map $\mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \to (\mathrm{V},\mathrm{M},\mathrm{J})$ is said to be **$\mathbf{nIgsemi}^*$-open map** if $\mathrm{f}(\mathrm{G})$ is $\mathrm{nIgsemi}^*$-open in $(\mathrm{V},\mathrm{M},\mathrm{J})$ for every nano open set G in $(\mathrm{U},\mathrm{N},\mathrm{I})$.

Example 3.2 Let $\mathrm{U} = \{1,2,3,4\}$, the approximation space $\mathrm{U}/\mathcal{R} = \{\{1\},\{2\},\{3,4\}\}$, $\mathrm{X} = \{3\} \subseteq \mathrm{U}$ with the ideal $\mathrm{I} = \{\varphi,\{2\},\{1,2\}\}$. The nano topology defined by U is $\tau_{\mathrm{R}}(\mathrm{X}) = \{\mathrm{U},\varphi,\{3,4\}\}$and $\mathrm{nIgsemi}^*$-open sets are{ $\mathrm{U},\varphi,\{1\},\{2\},\{3\},\{4\},\{3,4\},\{1,4\},\{2,4\},\{2,3\},\{1,3\},\{2,3,4\},\{1,3,4\}$.
Assume $\mathrm{V} = \{1,2,3,4\}$, the approximation space $\mathrm{V}/\mathcal{R} = \{\{1\},\{4\},\{2,3\}\}$, $\mathrm{Y} = \{1,4\} \subseteq \mathrm{V}$ with the ideal $\mathrm{J} = \{\varphi,\{1\}\}$. The nano topology defined by V is $\Omega_{\mathrm{R}}(\mathrm{Y}) = \{\mathrm{V},\varphi,\{1,4\}\}$ and $\mathrm{nIgsemi}^*$-open sets are $\{\mathrm{V},\varphi,\{1\},\{2\},\{3\},\{4\},\{1,2\},\{3,4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\}\}$.
Let the function $\mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \to (\mathrm{V},\mathrm{M},\mathrm{J})$ can be defined as identity function. Then f is $\mathrm{nIgsemi}^*$-open map.

Theorem 3.3 If $\mathrm{f}\colon(\mathrm{U},\mathrm{N},\mathrm{I}) \to (\mathrm{V},\mathrm{M},\mathrm{J})$ is an n-open map then the function f is a $\mathrm{nIgsemi}^*$ -open map.

Proof: Assume G is any nano open set in U. Then $f(G)$ is n-open in V because the function f is n-open map. Every n-open set is known to be $nIgsemi^*$-open. Then $f(G)$ is $nIgsemi^*$-open in V. As a result, f is a $nIgsemi^*$ -open map.

Remark 3.4 The following example demonstrates that the converse of the above theorem is not true.

Example 3.5 Consider the universal set $U = \{1,2,3,4\}$, the approximation space $U/\mathcal{R} = \{\{1\},\{3\},\{2,4\}\}, X = \{1,2\} \subseteq U$ with the ideal $I = \{\varphi,\{1\}\}$. The nano topology defined by U is $\tau_R(X) = \{U,\varphi,\{1\},\{2,4\},\{1,2,4\}\}$ and $nIgsemi^*$-open sets are $\{\{1\},\{2\},\{4\},\{1,2\},\{2,4\},\{1,4\},\{2,3,4\},\{1,2,4\},U,\varphi\}$. Let $V = \{1,2,3,4\}$, the approximation space $V/\mathcal{R} = \{\{1\},\{2\},\{3,4\}\}$, $Y = \{\{2\},\{4\}\} \subseteq V$ with the ideal $J = \{\varphi,\{1\}\}$. The nano topology defined by V is $\Omega_R(Y) = \{V,\varphi,\{2\},\{3,4\},\{2,,3,4\}\}$and $nIgsemi^*$-open sets are $\{\{2\},\{3\},\{4\},\{3,4\},\{2,4\},\{2,3\},\{2,3,4\},V,\varphi\}$. The function $f:(U,N,I) \to (V,M,J)$ is defined as $f(1) = 3, f(2) = 2, f(3) = 1, f(4) = 4$ is a $nIgsemi^*$-open map. But its not nano open map since $\{2,4\} \notin \Omega_R(Y)$

Theorem 3.6 Every $nIgsemi^*$-open map is nIg-open map but not conversely.

Proof: Let $f:(U,N,I) \to (V,M,J)$ be a $nIgsemi^*$-open map and assume G is n-open set in U. Every n-open set is known to be $nIgsemi^*$-open. Therefore $f(G)$ is $nIgsemi^*$-open in V. Then $f(G)$ is nIg -open in V because every $nIgsemi^*$-open set is nIg-open. As a result, f is a nIg-open map.

Example 3.7 Let $U = \{1,2,3,4\}$, the approximation space $U/\mathcal{R} = \{\{1\},\{2\},\{3,4\}\}$, $X = \{3\} \subseteq U$ with the ideal $I = \{\varphi,\{2\},\{1,2\}\}$. The nano topology defined by U is $\tau_R(X) = \{U,\varphi,\{3,4\}\}$ and $nIgsemi^*$-open sets are{ $U,\varphi,\{1\},\{2\},\{3\},\{4\},\{3,4\},\{1,4\},\{2,4\},\{2,3\},\{1,3\},\{2,3,4\},\{1,3,4\}$.
Let $V = \{1,2,3,4\}$, with $V/R = \{\{3\},\{4\},\{1,2\}\}$ and $Y = \{2,4\}$. Then the nano topology $\Omega_R(Y) = \{\phi,V,\{3\},\{4\},\{1,2,4\}\}$ and $J = \{\phi,\{1\}\}$.Then $nIgsemi^*$-open sets are $\{\phi,V,\{1\},\{2\},\{3\},\{4\},\{3,4\},\{1,4\},\{2,4\},\{2,3\},\{1,3\},\{2,3,4\},\{1,3,4\},\{1,2,4\}\}$ and nIg-open sets are $\{\phi,V,\{1\},\{2\},\{3\},\{4\},\{1,2\},\{3,4\},\{1,4\},\{2,4\},\{2,3\},\{1,3\},\{2,3,4\},\{1,3,4\},\{1,2,4\}\}$. The function $f:(U,N,I) \to (V,M,J)$ is defined as $f(1) = 3, f(2) = 4, f(3) = 1, f(4) = 2$ is a nIg-open map. But its not $nIgsemi^*$-open map since $\{1,2\} \notin \tau_R(X)$.

Theorem 3.8 Every ng-open map is $nIgsemi^*$-open map .

Proof: : Let $f:(U,N,I) \to (V,M,J)$ be a ng-open map and let G be any nano open set in U. Every n-open set is known to be ng-open. Therefore $f(G)$ is ng-open in V. Then $f(G)$ is $nIgsemi^*$ -

open in V because every ng-open set is $nIgsemi^*$-open. Hence f is $nIgsemi^*$-open map.

Remark 3.9 The following example demonstrates that the converse of the above theorem is not true.

Example 3.10 Consider the universal set $U = \{1,2,3,4\}$, the approximation space $U/\mathcal{R} = \{\{1\},\{3\},\{2,4\}\}, X = \{1,2\} \subseteq U$ with the ideal $I = \{\varphi,\{1\}\}$. The nano topology defined by U is $\tau_R(X) = \{U,\varphi,\{2\},\{1,2\},\{2,3,4\}\}$ and $nIgsemi^*$-open sets are $\{\{1\},\{2\},\{4\},\{1,2\},\{2,4\},\{1,4\},\{2,3,4\},\{1,2,4\},U,\varphi\}$. Let $V = \{1,2,3,4\}$, with $V/R = \{\{2\},\{4\},\{1,3\}\}$ and $Y = \{1,4\}$. Then the nano topology $\Omega_R(Y) = \{\ \phi, V,\{4\},\{1,3\},\{1,3,4\}\}$ and $J = \{\ \phi,\{4\}\}$.Then $nIgsemi^*$-open sets are $\{\phi,V,\{1\},\{3\},\{4\},\{3,4\},\{1,4\},\{1,3\},\{1,3,4\},\{1,2,3\}\}$ and ng-open sets are $\{\phi,V,\{1\},\{3\},\{4\},\{3,4\},\{1,4\},\{1,3\},\{1,3,4\}\}$. The function $f : (U,N,I) \to (V,M,J)$ is defined as $f(1) = 4, f(2) = 3, f(3) = 1, f(4) = 1$ is a $nIgsemi^*$-open map. But its not ng-open map since $\{1,2,3\} \notin \Omega_R(Y)$.

Remark 3.11 The following example demonstrates that the composition of two $nIgsemi^*$-open maps need not be a $nIgsemi^*$-open map.

Example 3.12 Let $U = \{1,2,3,4\}$, the approximation space $U/\mathcal{R} = \{\{1\},\{2\},\{3,4\}\}, X = \{3\} \subseteq U$ with the ideal $I = \{\varphi,\{2\},\{1,2\}\}$. The nano topology defined by U is $\tau_R(X) = \{U,\varphi,\{3,4\}\}$and $nIgsemi^*$-open sets are\{ $U,\varphi,\{1\},\{2\},\{3\},\{4\},\{3,4\},\{1,4\},\{2,4\},\{2,3\},\{1,3\},\{2,3,4\},\{1,3,4\}$. Let $V = \{1,2,3,4\}$, the approximation space $V/\mathcal{R} = \{\{1\},\{4\},\{2,3\}\}$, $Y = \{1,4\} \subseteq V$ with the ideal $J = \{\varphi,\{1\}\}$. The nano topology defined by V is $\Omega_R(Y) = \{V,\varphi,\{1,4\}\}$ and $nIgsemi^*$-open sets are $\{V,\varphi,\{1\},\{2\},\{3\},\{4\},\{1,2\},\{3,4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\}\}$. Let $W = \{1,2,3,4\}$, the approximation space $W/\mathcal{R} = \{\{2\},\{4\},\{1,3\}\}, Z = \{1,4\} \subseteq W$ with the ideal $K = \{\varphi,\{4\}\}$. The nano topology defined by W is $\Psi_R(Z) = \{W,\varphi,\{4\},\{1,3\},\{1,3,4\}\}$and $nIgsemi^*$-open sets are $\{W,\varphi,\{1\},\{3\},\{4\},\{1,2,3\},\{1,3,4\},\{1,3\},\{1,4\},\{3,4\}\}$. Let the function $f : U \to V$ can be defined by $f\,(1) = 1;\ f(2) = 4;\ f(3) = 3;\ f(4) = 2$ and $g : V \to W$ is defined as identity function. Here the functions f and g are $nIgsemi^*$-open map. Since $(g \circ f\,)(\{3,4\}) = g(f(\{3,4\}) = g(\{2,3\}) = \{2,3\}$ and here the $\{2,3\}$ is not a $nIgsemi^*$-open set in W, $g \circ f$ is not $nIgsemi^*$-open map.

Theorem 3.13 If $f: (U,N,I) \to (V,M,J)$ is nano open and $g: (V,M,J) \to (W,O,K)$ is $nIgsemi^*$-open, then $(g \circ f): (U,N,I) \to (W,O,K)$ is $nIgsemi^*$ -open.

Proof: Let G be any nano open set in U. Now $f(G)$ is nano open in V because $f:(U,N,I) \to (V,M,J)$ is nano open map. Also $g(f(G))$ is $nIgsemi^*$-open in W, since $g:(V,M,J) \to (W,O,K)$ is $nIgsemi^*$ -open. Therefore $(g \circ f):(U,N,I) \to (W,O,K)$ is a $nIgsemi^*$-open map.

Theorem 3.14 A surjective mapping $f:(U,N,I) \to (V,M,J)$ is $nIgsemi^*$-open if and only if, there exists a $nIgsemi^*$-closed set K of V containing S such that $f^{-1}(K) \subset F$ for any subset S of V and for any nano closed set F of U containing $f^{-1}(S)$.

Proof: Necessity: Assume that f is $nIgsemi^*$-open. Let U containing $f^{-1}(S)$ and let $S \subset V$, F be any n-closed set of U. In other words, $F \supset f^{-1}(S)$ implies $f(F) \supset S$. Put $K = f(F)$. When this occurs, K is a $nIgsemi^*$-closed set of V containing S such that $f^{-1}(K) \subset F$.

Sufficiency: Assume G is any nano open set of U and B be any subset of (V, M, J). Put $B = (f(G))^c$. In this case, $f^{-1}(B) \subset G^c$ and G^c is nano closed set in U. By hypothesis, there exists a $nIgsemi^*$-closed set K of V such that $B \subset K$ and $f^{-1}(K) \subset G^c$. Now $f^{-1}(K) \subset G^c$ implies $G \subset (f^{-1}(K))^c = f^{-1}(K^c)$. Hence we obtain $f(G) \subset K^c$------------(1). Now $B \subset K$ implies $(f(G))^c \subset K$. Thus $f(G) \supset K^c$------------(2). From (1) and (2), $f(G) = K^c$. Since K^c is $nIgsemi^*$-open, $f(G)$ is $nIgsemi^*$ -open in V. This implies that f is $nIgsemi^*$-open.

Theorem 3.15 A map $f:(U, N, I) \to (V, M, J)$ is a $nIgsemi^*$-open map if and only if $f(nIgsemi^*int(A)) \subseteq nIgsemi^*int(f(A))$ for each nano open set A in U.

Proof: Let A be a nano open set in U. Suppose that $f:(U, N, I) \to (V, M, J)$ is a $nIgsemi^*$-open map. Then $f(A)$ is $nIgsemi^*$-open in V. Therefore $f(A) = nIgsemi^*\text{-}int(f(A))$. Since $nIgsemi^*\text{-}int(A) \subseteq A$, $f(nIgsemi^*\text{-}int(A)) \subseteq f(A)$. Hence $f(nIgsemi^*\text{-}int(A) \subseteq nIgsemi^*(int(f(A))$.

Conversely, suppose A is an n-open set in U. Now $f(n\text{-}int(A) \subseteq f(nIgsemi^*int(A)) \subseteq nIgsemi^*(int(f(A))$. Since $n\text{-}Int(A) = A, f(A) \subseteq nIgsemi^*(Int(f(A))$--------------(1) From the definition of $nIgsemi^*$-int, $nIgsemi^*(Int(f(A)) \subseteq f(A)$-------------(2). From (1) and (2), $nIgsemi^*(Int(f(A)) = f(A)$. Therefore $f(A)$ is $nIgsemi^*$-open in V. Hence $f:(U, N, I) \to (V, M, J)$ is a $nIgsemi^*$-open map.

Theorem 3.16 Let $(U, N, I), (V, M, J)$ and (W, O, K) be any three nano ideal topological spaces. Let $f:(U, N, I) \to (V, M, J)$ and $g:(V, M, J) \to (W, O, K)$ be any two maps. Then the following statements are true.

1. If $(g \circ f)$ is a $nIgsemi^*$ -open map and f is nano continuous and, then g is a $nIgsemi^*$-open map.

2. If g is $\mathrm{nIgsemi}^*$-continuous and $(g \circ f)$ is an nano open map, then f is $\mathrm{nIgsemi}^*$-open map.

Proof: 1. Let's assume G be a nano open set in V. Then in U, $f^{-1}(G)$ is nano open . Given that $(g \circ f)$ is an $\mathrm{nIgsemi}^*$-open map, $(g \circ f)\left(f^{-1}(G)\right) = g\left(f\left(f^{-1}(G)\right)\right) = g(G)$ is $\mathrm{nIgsemi}^*$-open in W. Thus g is a $\mathrm{nIgsemi}^*$-open map.

2. Let's assume A be any n-open set in U. Since $(g \circ f)$ is n-open map, $g(f(A))$ is a nano open set in W. Also since g is $\mathrm{nIgsemi}^*$-continuous, $g^{-1}(g\left(f(A)\right) = f(A)$ is a $\mathrm{nIgsemi}^*$-open set in V. As a result f is $\mathrm{nIgsemi}^*$-open map.

Conclusion: In this research, we introduced two novel types of maps in nano ideal topological spaces, referred to as $\mathrm{nIgsemi}^*$-closed map and $\mathrm{nIgsemi}^*$-open map. These sets' fundamental characteristics and properties are provided.

References

[1] Arul Jesti, J & Joycy Renuka, J, 2020, 'Nano Ideal Generalised Semi*-Closed Sets in Nano Ideal Topological Spaces', Turkish Journal of Computer and Mathematics Education, vol 11, issue 3, pp. 2176-2184.

[2] Arul Jesti, J & Joycy Renuka, J, 2022, 'Nano Ideal Generalised Semi*-Continuous and Irresolute Functions in Nano Ideal Topological Spaces', Journal of Fundamental & Comparative Research, vol VIII, issue 12, no. 11, pp. 23-30.

[3] Bhuvaneswari, K & Mythili Gnanapriya, K 2014, 'Nano generalized closed sets in nano topological spaces', International Journal of Scientific and Research Publications, vol. 4, no. 5, pp. 1–3.

[4] Ganesan, S, 'nIg-Homeomorphism in nano ideal topological spaces' (Communicated).

[5] Lellis Thivagar, M & Carmel Richard 2013, 'On nano continuity', International Journal of Mathematics and statistics invention, vol. 1, no. 1, pp. 31 - 37.

[6] Parimala, M, Noiri, T & Jafari, S, 'New types of nano topological spaces via nano ideals'. Available from: https://www.researchgate.net/publication/315892279.