Journal of Namibian Studies, 35 (2023): 1-17 ISSN: 2197-5523

Special Issue On Engineering, Technology And Sciences

The Application Of Matlab® Live Script And Simulink In The Electrical Engineering Mathematics Classroom

Michelle Ellis Erasmus

Department of Mathematical and Physical Sciences,
Central University of Technology, Bloemfontein,
Free State, South Africa
mellis@cut.ac.za

Abstract

The Central University of Technology is in possession of a MATLAB® total academic headcount license, enabling all electrical engineering students to access MATLAB® and Simulink online for the duration of their studies. The student access and the limitations associated with paper bounded calculations led to an investigation into the extent this software can be utilised in the electrical engineering mathematics classroom to enhance student adoption of mathematics and its extension to realistic applications. The Live script application within this software allows text, images and coding in an interactive student friendly script which is well suited to the adoption of electrical engineering mathematics content by students as theory, examples, problem solution visualization and experimentation opportunity are present in a single script. Solution visualization and experimentation with realistic applications are important in electrical engineering field but is not possible in the traditional mathematics paper-bound delivery. Simulink, a graphic block diagram application within MATLAB®, was also investigated for its mathematics applications as this would appeal to the visual-understanding inclined student as well as prepare students for the application toolboxes they will use at a later stage of their studies. This paper will demonstrate how Live script and Simulink was turned into a mathematics textbook by sharing the student experience in determining the Fourier series for a periodic function and solving an RLC circuit for both a paper-suitable and paper-unsuitable application. Mathematics has evolved to an applications subject well received by its students at the Central University of Technology, tell everyone!

Keywords: Live script, Simulink, Interactive mathematics textbook.

1. Introduction

Interventions and innovations in mathematics delivery to increase the employability through skills enhancement of engineering students, is on the increase [2,6]. Students are expected to develop critical skills such as problem-solving, communication, teamwork, and leadership quickly and effectively [5,7].

Before the inclusion of MATLAB® software in 2019, mathematics delivery was in the traditional way with students accessing content notes or a prescribed online textbook from their student platform and attending mathematics classes during which time theory was detailed, examples were demonstrated, and students applied their knowledge to solving problems on paper. This restricted the applications to problems suited to paper solutions which often did not depict real world applications as the focus was primarily on the method. A typical example is the application of the Laplace transform or the complimentary and particular solution method in solving the RLC circuit $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V(t)$ where L is the inductance, R the resistance, C the capacitance, q the charge in the circuit and V the voltage power source [1]. Before the inclusion of software, the choices for L, R and C were restricted to whole numbers and the choices of $V(t)$ to functions such as e^t, t or $\sin(t)$ which results in a paper manageable but unrealistic outcome students can't relate to in comparison to their findings in other electrical subjects. These choices are still essential during method demonstrations but in an application question, which is expected in engineering mathematics, the circuit parameters and voltage source should reflect a more relatable circuit students would encounter in other electrical subjects such as the circuit shown in Figure 1 which has equation

$$650\times10^{-3}\frac{d^2q}{dt^2}+250\frac{dq}{dt}+\frac{1}{1.5\times10^{-6}}q=120\sqrt{2}\sin(2\pi60t)$$

. This equation is desirably more complicated to solve on paper and would not have been included in mathematics content before the introduction of Live script and Simulink. Another feature which was not possible in the pre software years is the reversal of a desired output back to the system components it generated from through experimentation. Reverse engineering brings better understanding to the underlying processes for many engineering aspects. With the inclusion of the software, students can tune parameters in the code to deliver the desired output from which the equation can be modeled and solved algebraically with the methods instructed. It is desirable that engineering mathematics is not perceived by students as a standalone subject due to the unrelatable problems being solved [4,8]. The use of Live script as a textbook for engineering mathematics is not a common occurrence at universities of technology in South Africa and has upgraded mathematics at the Central university of technology to an application subject status.

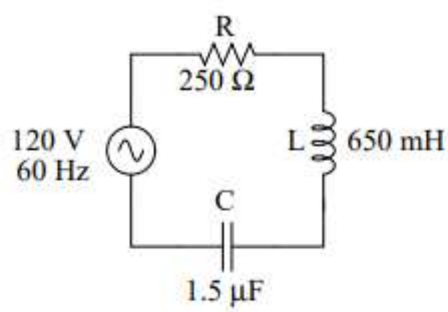


Fig.1 RLC circuit from Electronics textbook (https://www.allaboutcircuits.com/textbook/alternating-current/chpt-5/series-r-l-and-c/)

2. Methodology

Live (.mlx) scripts together with Simulink (.slx) diagrams were prepared containing theory, examples, and code for all pre graduate electrical engineering mathematics subjects between 2020 and 2021. They cover all traditional mathematics topics such as wave theory, vector theory , linear algebra, complex numbers, differential calculus, integration, basic statistics, Fourier series, Laplace transform, state-space equations, solving of differential equations and numerical methods. As the Live scripts can toggle between text (with equation editor that includes LaTeX) and code and allows for the insertion of images, previous material could be utilized with the addition of coding of demonstrated examples. This compares to a text-book experience, such as can be found in the paperback [3], where MATLAB® code and mathematics content is blended except that one can't interact with a paperback. Students register and activate their Mathworks accounts and are then able to access MATLAB® online from any computer with access to the internet. Access to the Live script and Simulink platforms are found in the MATLAB® work-window ribbon. Live script .mlx chapters are available on the student platform (blackboard) for students to download and access through MATLAB® online. Should students require a hard copy for offline use, all .mlx files can be exported to either a word or pdf document.

Each chapter in the script series initiates with required theory followed by worked out examples and the code required to duplicate the outcome. As the code can be interacted with, students acquire coding basics and can experiment with different values. As students are required to perform hand calculations during assessments, sufficient paper appropriate problems are included which students can also code and compare the outcome with their own solutions. Once students are confident in the reliability of the code, the chapter flows into assignments containing more realistic engineering problems, unsuited to hand calculations. The latter ability addresses the shortcoming of electrical engineering relatable content in the traditional paper delivery of mathematics. The following two examples demonstrate the student experience in the flow from hand-to-code calculations in the electrical engineering mathematics classroom.

3. Calculating the Fourier series for a given periodic function

The calculation of the Fourier series (FS) for a periodic signal is a technique for expressing a periodic function in terms of sinusoids [1]. Figure 2 shows a worked out example in the .mlx script in the calculation of the FS for the periodically defined triangular wave in the figure. The necessary theory containing the derivation of the integrals and integration techniques used in the calculation precede the worked-out example in the same script. Students are put through the hand calculation in steps ending in the calculations of the harmonics and expressing them in terms of sinusoids. In Figure 3 the text containing the example flows into the coding section where the code demonstrates the use of the unit step function or Heaviside function ($H(t)$) in defining the periodic interval, for example, if term A starts at $t = a$ and

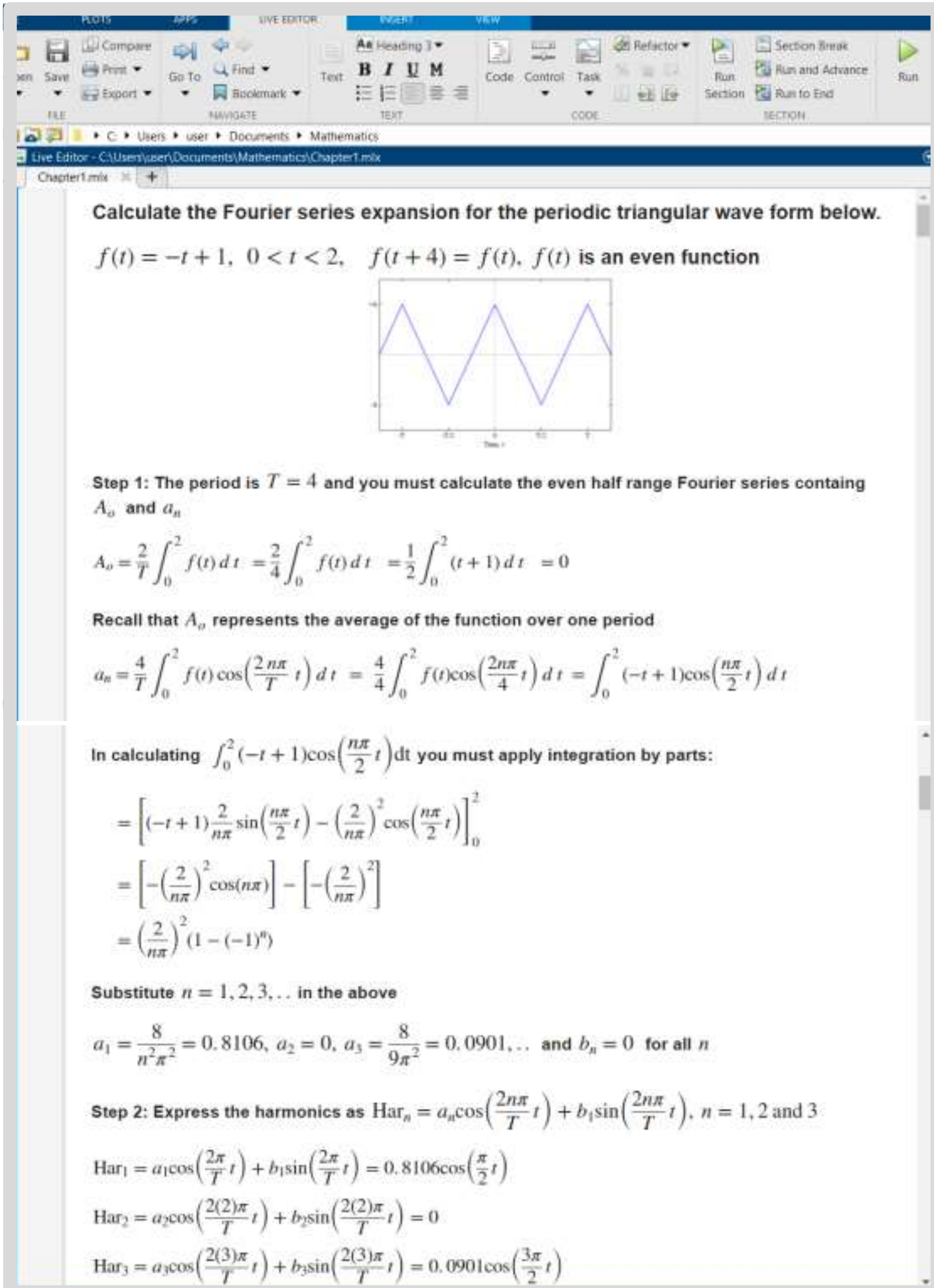
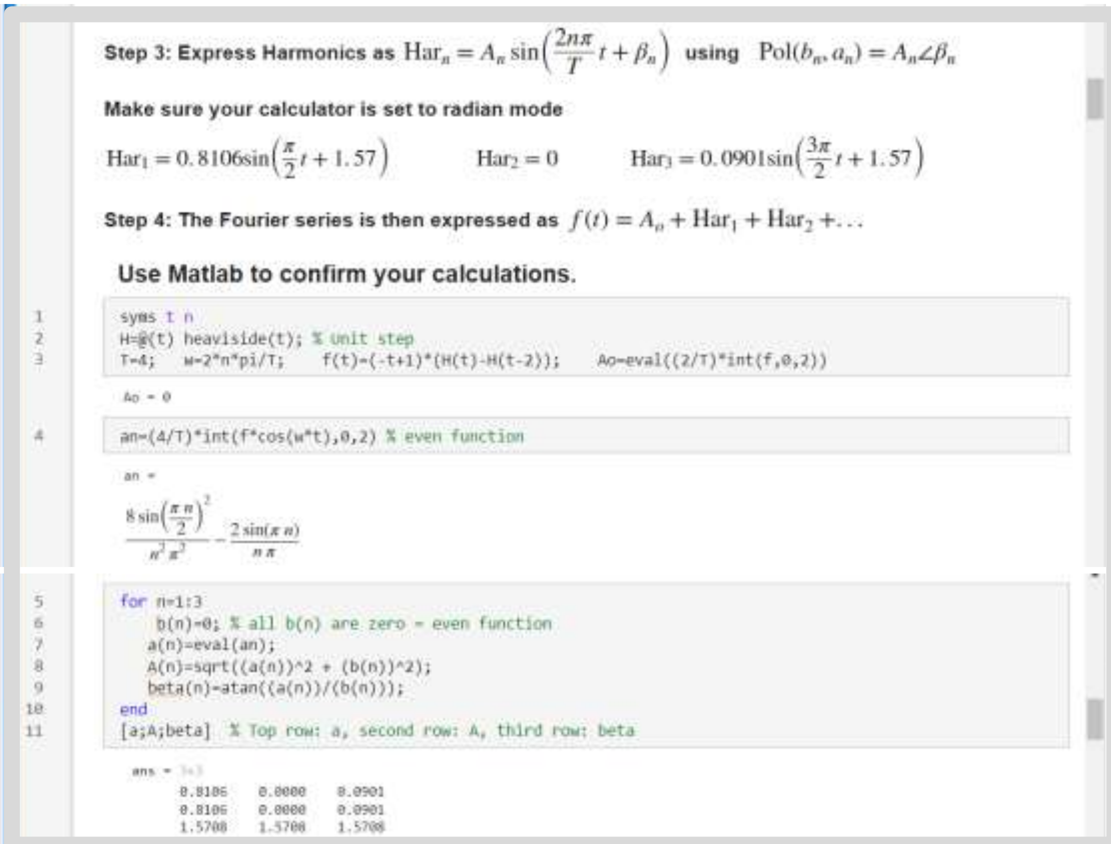


Fig.2 Fourier series calculation for the periodic triangular wave form.



Step 3: Express Harmonics as $\mathrm{Har}_n = A_n \sin\left(\frac{2n\pi}{T}t + \beta_n\right)$ **using** $\mathrm{Pol}(b_n, a_n) = A_n\angle\beta_n$

Make sure your calculator is set to radian mode

$\mathrm{Har}_1 = 0.8106\sin\left(\frac{\pi}{2}t + 1.57\right)$ $\quad \mathrm{Har}_2 = 0$ $\quad \mathrm{Har}_3 = 0.0901\sin\left(\frac{3\pi}{2}t + 1.57\right)$

Step 4: The Fourier series is then expressed as $f(t) = A_o + \mathrm{Har}_1 + \mathrm{Har}_2 + \ldots$

Use Matlab to confirm your calculations.

```
syms t n
H=@(t) heaviside(t); % unit step
T=4;   w=2*n*pi/T;    f(t)=(-t+1)*(H(t)-H(t-2));    Ao=eval((2/T)*int(f,0,2))
```

Ao = 0

```
an=(4/T)*int(f*cos(w*t),0,2) % even function
```

an =

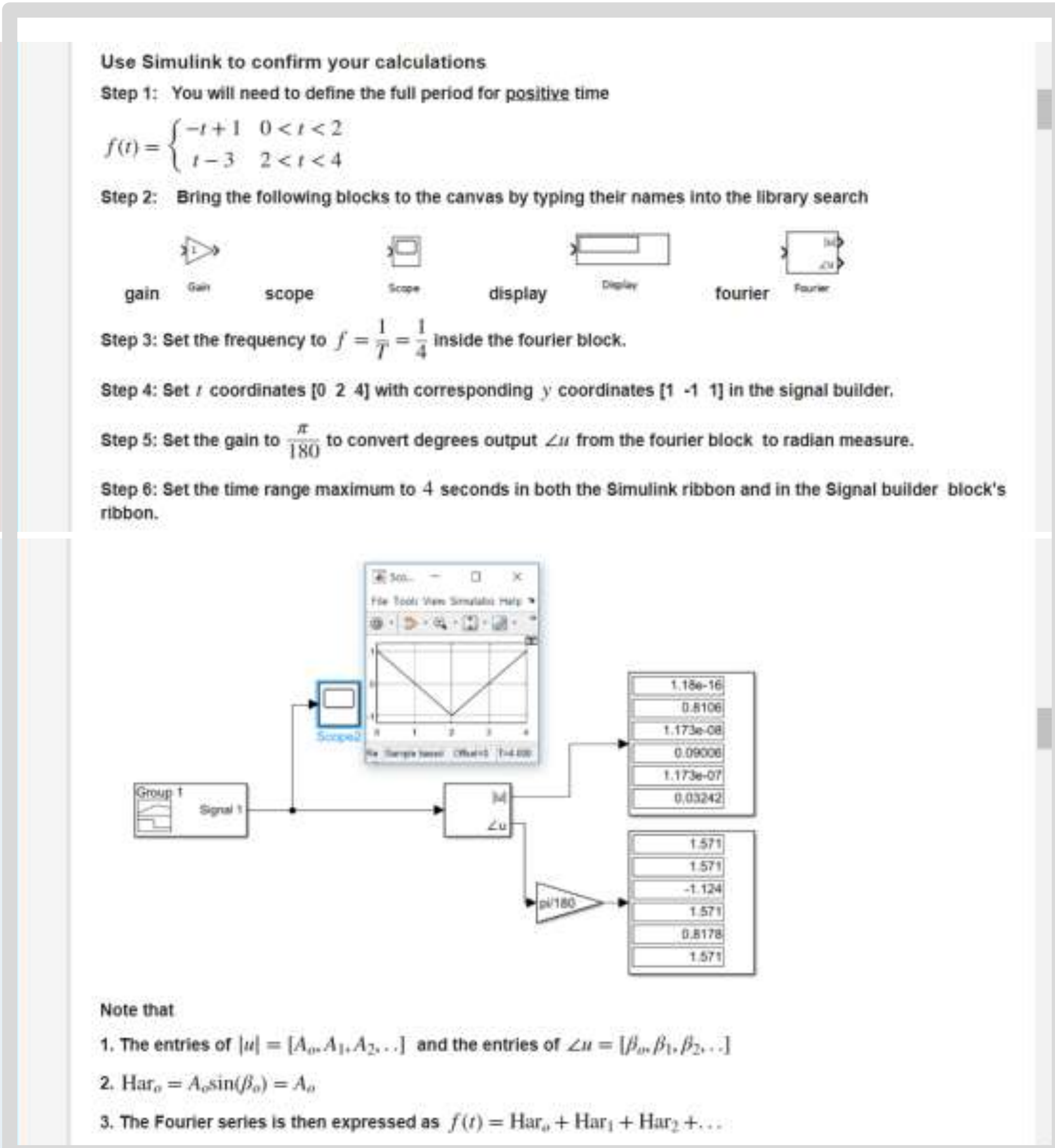
$$\frac{8\sin\left(\frac{\pi n}{2}\right)^2}{n^2\pi^2} - \frac{2\sin(\pi n)}{n\pi}$$

```
for n=1:3
    b(n)=0; % all b(n) are zero - even function
   a(n)=eval(an);
   A(n)=sqrt((a(n))^2 + (b(n))^2);
   beta(n)=atan((a(n))/(b(n)));
end
[a;A;beta]  % Top row: a, second row: A, third row: beta
```

```
ans = 3x3
    0.8106    0.0000    0.0901
    0.8106    0.0000    0.0901
    1.5708    1.5708    1.5708
```

Fig.3 Continuation of the calculation of the Fourier series for the triangular wave form.

ends at $t=b$, it is expressed as the switch combination $A(H(t-a)-H(t-b))$. Students can adapt the code and compare the output to their hand calculated solutions. With the freedom that comes with the coding skill, students can calculate the Fourier series for more complicated periodic functions such as $f(t)=\sqrt{1-t^2},\ 0<t<1,\ f(t+1)=f(t)$ which will result in integrals that can't be solved by hand. In addition to the code, students are also exposed to the Simulink platform which they access from the MATLAB® ribbon. This environment contains a toolbox dedicated to electrical applications and contains a Fourier block that calculates the FS coefficients given the description of the periodic function. The instructions are also contained in the live script and appears as shown in Figure 4. The required blocks are sourced from the Simulink library which is demonstrated during a classroom session. The signal-builder block is used to describe the periodic function as a set of coordinates describing vertices in positive time. The output of the Fourier block contains the amplitudes $|u|$ and the phase angles $\angle u$ (degrees) of the sinusoids. To convert the degree measure to radian measure, a gain block is used with factor set to $\frac{\pi}{180}$. The output agrees with those calculated both by hand and the Live script code. The signal builder block is used when the input signal is linearly defined but the sources folder in the Simulink library contains a variety of source blocks associated with the electrical field. Simulink captivates its student audience on account of its 'fun' structure with students readily complying to interact with Simulink and experimenting in this uncomplicated environment that elegantly executes mathematics.



Use Simulink to confirm your calculations

Step 1: You will need to define the full period for positive time

$$f(t)=\begin{cases}-t+1 & 0<t<2\\ t-3 & 2<t<4\end{cases}$$

Step 2: Bring the following blocks to the canvas by typing their names into the library search

gain Gain scope Scope display Display fourier Fourier

Step 3: Set the frequency to $f=\frac{1}{T}=\frac{1}{4}$ inside the fourier block.

Step 4: Set t coordinates [0 2 4] with corresponding y coordinates [1 -1 1] in the signal builder.

Step 5: Set the gain to $\frac{\pi}{180}$ to convert degrees output $\angle u$ from the fourier block to radian measure.

Step 6: Set the time range maximum to 4 seconds in both the Simulink ribbon and in the Signal builder block's ribbon.

Note that

1. The entries of $|u|=[A_o,A_1,A_2,\ldots]$ and the entries of $\angle u=[\beta_o,\beta_1,\beta_2,\ldots]$
2. $\mathrm{Har}_o=A_o\sin(\beta_o)=A_o$
3. The Fourier series is then expressed as $f(t)=\mathrm{Har}_o+\mathrm{Har}_1+\mathrm{Har}_2+\ldots$

Fig.4 Instructions for the Simulink diagram in calculating the Fourier series of the triangular wave form.

4. Calculating the output of an RLC smoothing circuit

The example in Figure 5 which is continued in Figure 6, demonstrates the application of the Laplace transform in solving a second order differential equations resulting from the application of Kirchoff's rule to the voltage drop across the components in a closed RLC circuit containing a resistor (R), capacitor (C), Inductor (L) and voltage source V_s. The Laplace transform is a technique that turns a differential equation into an algebraic equation which is easier to manipulate towards its solution [1]. The circuit components in this example are chosen to smooth the output of a digital pulse which is piecewise defined. Different choices of these components will have different effects on the output which students can experimented with to tune a desired outcome. As the voltage source is piecewise defined, the Laplace method is the best solution tool but does require a lot of hand calculations. This example is preceded by the necessary theory and simple examples not relatable to real world applications before moving on to more realistic content. The piecewise input voltage source V_s in this example can be expressed as the sum of its parts defined by $V_s = V_1 + V_2 + V_3$ where the Heaviside function is used to turn parts on and off as shown below:

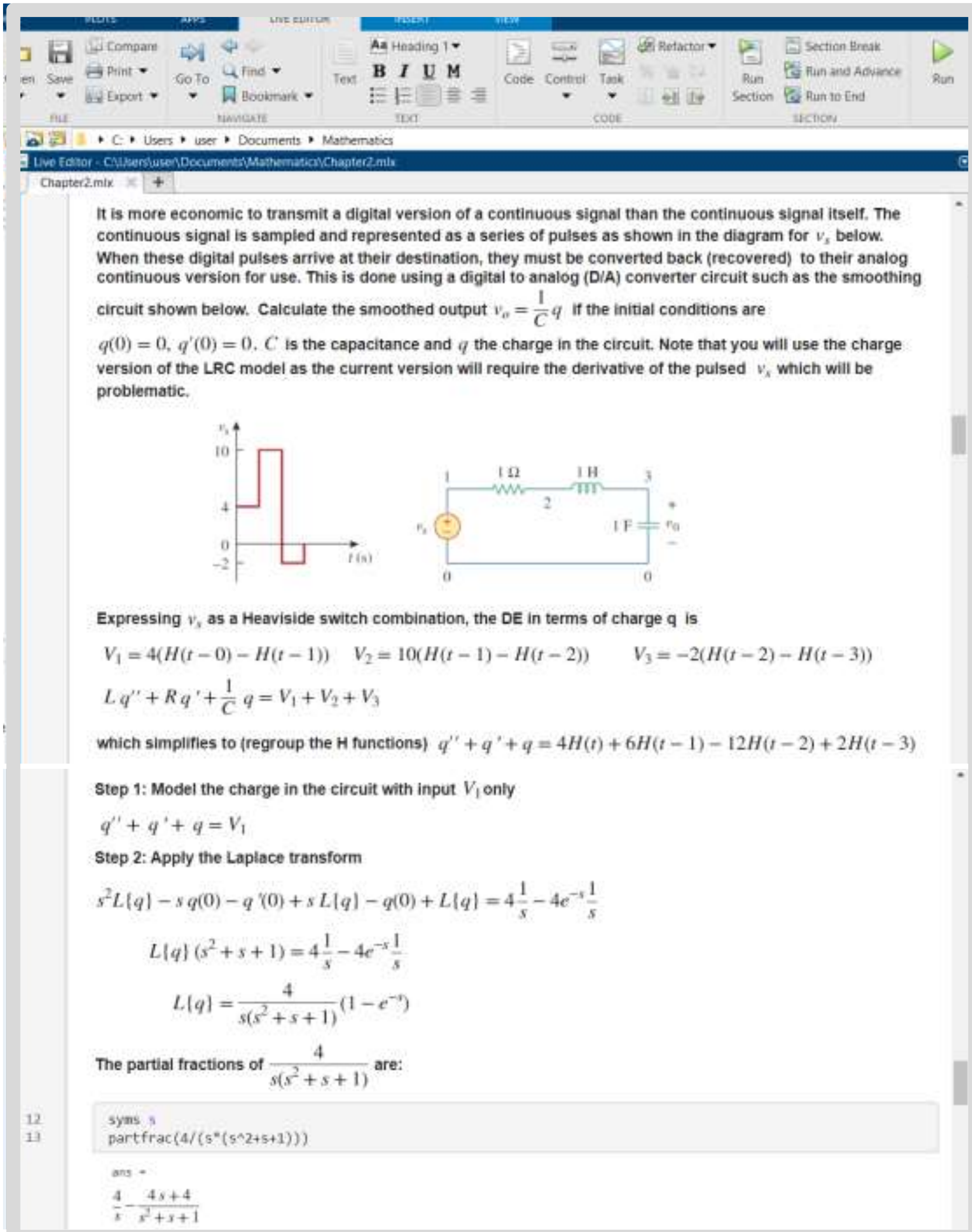
$V_1 = 4(H(t-0) - H(t-1))$,

$V_2 = 10(H(t-1) - H(t-2))$,

$V_3 = -2(H(t-2) - H(t-3))$

(1)

The circuit is modeled by $L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{1}{C}q = V_s$ which simplifies to $q'' + q' + q = V_1 + V_2 + V_3$ for the given circuit components $R = 1\,\Omega,\ L = 1\,H,\ C = 1\,F$. As the calculations will exceed what students can afford in a restricted time frame, only one input voltage component can be included during hand calculations. The solution steps are detailed in Figure 5 and 6 for the inclusion of only V_1 which shows the complexity of the calculations. The calculations are followed by the required code for the truncated problem and the smoothed output graphically illustrated in Figure 7. Students can adapt the code to include all three voltage components and observe the change in the smoothed output. Although they could not calculate the complete solution by hand, the Laplace mechanism is understood, and students can appreciate the complete solution they coded themselves. In addition to the code, the Simulink equivalent is also detailed in the script as shown in Figure 7. Students can build the diagram using the prescribed blocks, practice coding the signal builder and follow the logic of how the differential equation is modeled using the integrator block $\boxed{\frac{1}{s}} \rightarrow \int$ to integrate $q'' \rightarrow \int \rightarrow q' \rightarrow \int \rightarrow q$ where q is the solution and output to the display block. The diagram models the altered equation $q'' = \left(\frac{1}{L}\right)V_s - \left(\frac{R}{L}\right)q' - \left(\frac{1}{LC}\right)q$ where the coefficients of q' and q are respectively captured in the gain blocks Gain and Gain1 in Figure 7. These terms flow back to combine with V_s in the summation block to form the right hand side of the altered equation. Students can tune the parameters of the Simulink diagram to pursuit a specific outcome or they can change the input voltage. The source folder in the Simulink library contains



It is more economic to transmit a digital version of a continuous signal than the continuous signal itself. The continuous signal is sampled and represented as a series of pulses as shown in the diagram for v_s below. When these digital pulses arrive at their destination, they must be converted back (recovered) to their analog continuous version for use. This is done using a digital to analog (D/A) converter circuit such as the smoothing circuit shown below. Calculate the smoothed output $v_o = \frac{1}{C} q$ if the initial conditions are $q(0) = 0,\ q'(0) = 0$. C is the capacitance and q the charge in the circuit. Note that you will use the charge version of the LRC model as the current version will require the derivative of the pulsed v_s which will be problematic.

Expressing v_s as a Heaviside switch combination, the DE in terms of charge q is

$$V_1 = 4(H(t-0) - H(t-1)) \quad V_2 = 10(H(t-1) - H(t-2)) \quad V_3 = -2(H(t-2) - H(t-3))$$

$$L\,q'' + R\,q' + \frac{1}{C}\,q = V_1 + V_2 + V_3$$

which simplifies to (regroup the H functions) $q'' + q' + q = 4H(t) + 6H(t-1) - 12H(t-2) + 2H(t-3)$

Step 1: Model the charge in the circuit with input V_1 only

$$q'' + q' + q = V_1$$

Step 2: Apply the Laplace transform

$$s^2 L\{q\} - s\,q(0) - q'(0) + s\,L\{q\} - q(0) + L\{q\} = 4\frac{1}{s} - 4e^{-s}\frac{1}{s}$$

$$L\{q\}\,(s^2 + s + 1) = 4\frac{1}{s} - 4e^{-s}\frac{1}{s}$$

$$L\{q\} = \frac{4}{s(s^2 + s + 1)}(1 - e^{-s})$$

The partial fractions of $\frac{4}{s(s^2+s+1)}$ are:

```
syms s
partfrac(4/(s*(s^2+s+1)))
```

ans =

$$\frac{4}{s} - \frac{4s+4}{s^2+s+1}$$

Fig.5 Using the Laplace transform to calculate the output of an RLC circuit.

many circuit-applicable input blocks students can experiment with. In typical assignment students are requested to build a Simulink source such as V_s by instead using the right combination of amplitude tuned Step blocks (H (t)) from the source folder to model V_s as the combinations shown in (1). This challenges students to be creative when solving problems which is a very useful skill in engineering.

Step 3: Identify the time functions $\frac{4}{s}$ and $\frac{4s+4}{s^2+s+1}$ **came from. First fraction:** $\frac{4}{s} = L\{4\}$

Second fraction: Complete the square $s^2+s+1 = (s+0.5)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$$\frac{4s+4}{(s^2+s+1)} = \frac{4s+4}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 4\left(\frac{s+0.5-0.5+1}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$= 4\left(\frac{s+0.5}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{0.5}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$= 4\left(\frac{s+0.5}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{0.5}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$= 4\left(\frac{s+0.5}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{2(0.5)}{\sqrt{3}}\,\frac{\frac{\sqrt{3}}{2}}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$= 4\,L\left\{e^{-0.5t}\cos\left(\frac{\sqrt{3}}{2}t\right)\right\} + \frac{4}{\sqrt{3}}L\left\{e^{-0.5t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right\}$$

This brings the calculations to

$$\frac{4}{s} - \frac{4s+4}{s(s^2+s+1)} = L\left\{4 - 4e^{-0.5t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{4}{\sqrt{3}}e^{-0.5t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right\} = L\{f(t)\}$$

where $f(t) = 4 - 4e^{-0.5t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{4}{\sqrt{3}}e^{-0.5t}\sin\left(\frac{\sqrt{3}}{2}t\right)$

getting back to $L\{q\}$:

$$\begin{aligned} L\{q\} &= L\{f(t)\}(1-e^{-s}) \\ &= L\{f(t)\} - e^{-s}L\{f(t)\} \end{aligned}$$

Step 4: The equation is now fully linear in L **and we can take the inverse Laplace transform**

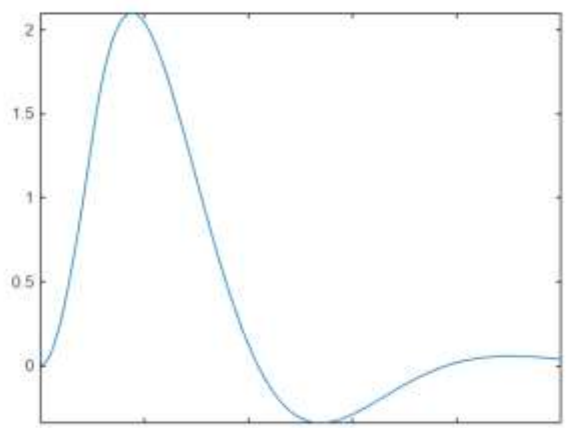
$$q(t) = 4f(t) - 4f(t-1)H(t-1) \qquad = \begin{cases} 4f(t) & 0 < t < 1 \\ 4f(t) - 4f(t-1) & t > 1 \end{cases}$$

Step 5: Expressing the solution in terms of output voltage over the capacitor, $v_o(t) = \frac{1}{C}q(t) = q(t)$ **as C=1.**

Fig.6 Continuation of the calculations using the Laplace transform for an RLC circuit.

Use Matlab to confirm your calculations and plot the output voltage

```
syms q(t)
H=@(t) heaviside(t);  V1=4*(H(t-0)-H(t-1));
dq=diff(q);   dq2=diff(q,2);
eq=dq2+dq+q==V1;
q=dsolve(eq,q(0)==0,dq(0)==0);
fplot(q,[0,10])
```



Use Simulink to determine the output for $V_s = V_1 + V_2 + V_3$

Step 1: Observe the differential equation as $q'' = -q' - q + V_s$

Step 2: Bring the following blocks to the canvas by typing their names into the library search

Step 2: Set t coordinates [0 1 1 2 2 3 3] with corresponding v coordinates [4 4 10 10 -2 -2 0] in the signal builder.

Step 3: Set the time range maximum to 4 seconds in both the Simulink ribbon and in the Signal builder block's ribbon.

Step 4: Set the initial conditions to 0 in both integrator blocks.

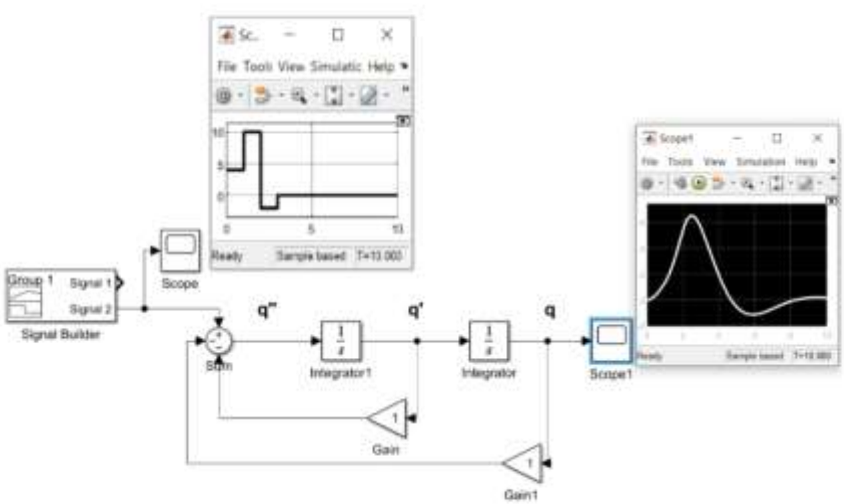


Fig.7 Simulating the RLC circuit output using Live script and Simulink.

5. Conclusion

The extension to relatable application simulations in the classroom is crucial in the study of electrical engineering subjects which includes mathematics. The inclusion of simulation software has become synonymous with the study of electrical engineering and is well represented in subjects with a high application component. With the CUT campus licence and the Live script textbook, students can to a large extent self-pace through mathematics content both on and off campus. In addition to their qualification, students also add a valuable coding skill to their skills basket which enhances their employability. The use of Live script and Simulink as a complete mathematics application textbook is a new intervention in the pursuit of addressing the applicability and relevance of mathematics content and it is the purpose of this paper to motivate other universities of technology in South Africa to explore this option.

6. References

[1] C.K. Alexander and M.N.O. Sadiku. Fundamentals of Electric Circuits, 4th Edition, ISBN 9780073529554, Chapters 7 and 8, pp.15-17, **(2009)**

[2] E. Cook. Practice-Based Engineering: Mathematical Competencies and Micro-Credentials. International Journal of Research in Undergraduate Mathematics Education, 7, pp.284–305 **(2021)**

[3] G. James and P. Dyke. Modern Engineering Mathematics (5th ed.). Harlow, United Kingdom, Pearson **(2018)**

[4] M. A. Majid, Z. A. Huneiti, W. Balachandran and Y. Balarabe. Matlab as a teaching and Learning tool for Mathematics: A Literature Review. International Journal of Arts & Sciences, CD-ROM. ISSN: 1944-6934, 6(3), pp.23–44 **(2013)**

[5] J. Mason, L. Burton and K. Stacey. Thinking Mathematically. Harlow, United Kingdom,
Pearson (2010)

[6] B. Pepin, R. Biehler and G. Gueudet. Mathematics in Engineering Education: a Review of the Recent Literature with a View towards Innovative Practices. International Journal of Research in Undergraduate Mathematics Education, 7, pp.163–188 **(2021)**

[7] A. H. Schoenfeld. Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grows (Ed.), NCTM Handbook of research on mathematics teaching and learning, pp. 334-370. New York, USA, Macmillan **(1992)**

[8] J. Smith, L. Johnson and M. Anderson. Perceptions of Mathematics as a standalone subject: The case of engineering students. Engineering Education Research Journal, 9(2), pp.45-60 **(2018)**

11.1. Journal Article

[1] C. D. Scott and R. E. Smalley, J. Nanosci. Nanotechnol. 3, 75 **(2003)**

11.2. Book

[2] H. S. Nalwa, Editor, Magnetic Nanostructures, American Scientific Publishers, Los Angeles **(2003)**

11.3. Chapter in a Book

[3] H. V. Jansen, N. R. Tas and J. W. Berenschot, in Encyclopedia of Nanoscience and Nanotechnology, Edited H. S. Nalwa, American Scientific Publishers, Los Angeles **(2004)**, Vol. 5, pp.163-275.

Author

Michelle Erasmus

Michelle Erasmus received her PhD in Applied Mathematics in 2013 from the University of the Free state. Her research interests include mathematical modeling and innovation in teaching and learning. She is currently a Lecturer at the Central University of Technology, Bloemfontein, South Africa